TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 754

CONTRIBUTION TO THE MUTUAL INTERFERENCE OF WING AND PROPELLER

By C. Wieselsberger

Abhandlungen aus dem Aerodynamischen Institut
an der Technischen Hoschschule Aachen
No. 13, 1933

Washington
September 1934

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SUMMARY

When the wing is outside of the slipstream the velocity field of the propeller modifies the air-stream direction on the wing (fig. 1) and through it the induced drag of the wing. On the other hand, the presence of the wing changes the inflow velocity of the propeller which stipulates a change in propeller efficiency.

A qualitative analysis shows that the change in power required for level flight due to changed induced drag equals the change in propeller output due to changed efficiency, when the original propeller is replaced by a substitute propeller producing the same slipstream as the uninfluenced propeller. Then the mutual interference appears only as inside force.

The change of induced wing drag due to the field of flow of the propeller was analyzed quantitatively. The field of flow of the propeller is represented by a uniform distribution of sinks over the propeller disk area, whose strength is determined by the increase of speed in the slipstream. The superposition of this sink flow on the basic flow reproduces the actual field of flow outside of the slipstream with close approximation. The computed change of induced wing drag is compared with experimental data and the agreement is found to be satisfactory.

I

In the analysis of mutual interference between wing and propeller, it is advisable to consider two cases which

*"Beitrag zur gegenseitigen Beeinflussung von Flügel und Luftschraube. Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule Aachen, no. 13, 1933, pp. 1-11.

require totally different methods of treatment. The first is characterized by the fact that the wing penetrates the slipstream. A part of the wing is in a zone of higher speed relative to the basic flow, with the result that the lift distribution (and consequently, the induced drag) are modified. This case does not lend itself readily to theoretical treatment, because the theory of the wing partially within a flow of higher speed, is beset with great difficulties and has therefore received comparatively little attention.*

The second case is characterized by the wing being wholly outside of the slipstream, where the speed changes are small enough to be negligible, so that only the changes in air-stream direction need to be considered, from which, then, the change of induced wing drag can be determined. Our analysis is limited to this latter case, since it actually occurs in many types of airplanes. Experiments made relative to this particular case (reference 1) have proved that, with the propeller, say, above the wing (fig. 1), the wing drag as well as the propeller thrust become less, compared to the undisturbed conditions (infinitely great distance between wing and propeller). Conversely, drag and thrust are higher if the propeller is below the wing. This is readily explained from elementary slipstream theory. It is seen that in the case of figure 1, the flow is upward at the point where the wing occurs as a result of the slipstream contraction. This puts the wing partly within an upwash with an ensuing smaller extraneously induced drag and diminished power required for level flight. Contrariwise, the speed above the wing is higher relative to the basic flow. Thus the effect of the propeller consists in a decrease in propeller thrust since the thrust is known to decrease as the speed increases. Now, the maximum theoretical propeller efficiency being proportional to the ratio of basic speed to rate of flow through propeller disk (reference 2), it is readily seen that the presence of the wing vitiates the maximum theoretical efficiency. The problem then narrows down to the quantitative determination of the increased power required for level flight resulting from the poorer propeller efficiency and the decrease in power required for level flight by virtue of the reduced wing drag. Airplane

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*L. Prandtl investigated a special case, namely, that of a wing extending through a jet but while the speed outside of the jet was zero. (Prandtl's Wing Theory, II, section 12.)

weight and flight speed are assumed constant. The equilibrium also stipulates constant lift. Hence the assumption that, at the points of the wing where, due to the upwash caused by the propeller, the angle of attack as well as the lift are modified, the original lift is reestablished by an appropriate change of angle of attack.

II

Before proceeding to the mathematical analysis, it may be informative to indicate briefly another analytical method pointed out by H. B. Helmbold (reference 3). This method, which affords a good insight into the existing phenomena, consists in analyzing the flow conditions remotely aft of the wing-propeller system. According to airfoil theory, the wing sheds vortices in the form of a sheet. In unit time this vortex system generates a new piece of length V and the power expended thereto exactly corresponds to the work of the induced drag $W_i V$. Thus the strength of the vortex system is a criterion for the magnitude of the induced drag. Since the lift A shall be constant, the self-induced drag,

$$W_i = \frac{A^2}{\pi q b^2}$$

(q = dynamic pressure, b = span) must be constant also, inasmuch as the flying speed was assumed constant. It follows that the strength of the vortex system with approximation of the propeller likewise remains perfectly the same. The upwash in which the wing finds itself in the case of figure 1, obviously causes a lower induced drag, but as this diminution is extraneously induced - that is, caused by the field of flow of the propeller - it has no influence on the strength of the vortex system. (If the efficiency relative to the efficiency with free running propeller remained unchanged, the decrease in induced wing drag would effect a climb of the airplane.) Moreover, the vortex system behind the wing is in nowise modified by the approach of the propeller.

The investigation of the propeller effect vicinal to the wing proceeds as follows: The slipstream is equally considered at great distance behind the propeller, and the original propeller is replaced by a substitute propeller producing a slipstream of equal section S and equal increase of speed v_a relative to the basic flow as the free running propeller. Because of the higher speed on the suction side of the wing, the propeller diameter must be decreased in the case of figure 1, or else the slipstream diameter at great distance behind the propeller would be greater on account of the greater volume of flow. The increase of speed v_a must be held constant by appropriate pitch setting. Thus, with S and v_a having the same values as the free running propeller, the substitute propeller has the same thrust and the same maximum theoretical efficiency as the free running propeller, for with constant flight speed through the jet section and with v_a the thrust and the efficiency are unequivocally defined. According to slipstream theory, thrust P and maximum theoretical efficiency η are written:

$$P = \rho\ S \left(V + \frac{v_a}{2}\right) v_a \tag{1}$$

$$\eta = \frac{V}{V + \frac{v_a}{2}} = \frac{2}{\sqrt{1 + c_s} + 1} \tag{2}$$

(c_s = load factor, ρ = air density)

Now the analysis of the vortex system far behind the wing has shown the induced drag and the power required for level flight $W_i\ V$ to remain unaltered, likewise thrust and propeller efficiency and consequently, propeller performance by the replacement with substitute propeller. From this follows that the combination wing-substitute propeller is in equilibrium since there is no change in flow energy at great distance behind this system. In other words, there is no mutual interference between wing and propeller with the formulated assumptions, which refutes the statement made at the beginning, that there is mutual interference. The explanation of this apparent inconsistency is that the mutual interference exists only as an internal force, and in such a way that the power required for level flight decreases as a result of the lower induced drag to the same amount as the propeller performance resulting from the poorer propeller efficiency, thus retaining the airplane equilibrium horizontally as before and outwardly evincing no reciprocal effect. It simply exists as internal force which would appear with the determination of the so-called hub-dynamometer thrust.

One conclusion of some interest to the airplane designer, may be drawn from the above. It pertains to the question of whether it would be more favorable to mount a propeller of given diameter (= diameter of free running propeller) on the suction side or on the pressure side of the wing. We have seen that, with the propeller on the suction side, the diameter of the substitute propeller must be reduced in comparison to that of the free-running propeller to insure the same slipstream diameter, or else the volume of flow will be greater than with the substitute propeller because of the greater propeller diameter. For equal thrust the load factor of the propeller thus becomes smaller and the maximum theoretical efficiency more propitious, according to equation (2). Consequently, the power input of the propeller is less for equal flight speed and the arrangement of the propeller above the wing constitutes an advantage.

A similar deduction reveals that, mounting the same propeller on the pressure side of the wing, stipulates a greater power input in order to maintain the original flight speed.

III

We have shown that the use of a substitute propeller does not alter the airplane equilibrium in horizontal direction (compared with the case of very great distance of wing-propeller). There is no outward appearance of mutual interference; it exists simply as an inside force.

However, as it is of interest to quantitatively determine the magnitude of these inside forces which exist as reciprocal effect between wing and propeller, we compute the change of induced drag due to the presence of the propeller, which affords the change in power required for level flight and at the same time the change in propeller performance, because both are equal in amount when replacing the original with the cited substitute propeller. Without the latter the conditions become somewhat more complicated, since both changes must be calculated separately. We abstain from this case because the changes are small, so that in first approximation the difference between change of power required for level flight and that of propeller performance may be disregarded.

The quantitative determination of changed induced drag due to the propeller is premised on the velocity field surrounding the propeller. K. Friedrichs has shown in an as yet unpublished report that a small load factor affords a practical approximate representation of the field of flow around a propeller with uniform thrust grading when the area of the propeller disk is covered evenly with sinks, so as to suck the fluid from both sides into the propeller disk. The rate of entry in the propeller disk is $v_a/2$, when v_a = increase of speed in slipstream relative to basic flow V. Allowing, in addition, the fluid to leave at the rear at speed v_a normal to the surface, the sink-flow and the basic flow together give a fair representation of the real flow including slipstream contraction (fig. 2). The fluid then passes steadily through the propeller disk at speed $V + v_a/2$, as stipulated in the slip stream theory.*

For the directional changes on the wing the radial components of the propeller flow alone are of influence and the sink-flow is the only one that contributes to it. As customary in airfoil theory the change of the absolute quantities of the speed is disregarded.

The origin of the coordinates is placed in the center of the propeller disk, its radius is a (fig. 3). The coordinate along the propeller axis in downstream direction is z and its corresponding normal radial component, r. No other coordinates are needed, because the flow is rotationally symmetrical relative to the thrust axis.

The first problem is to obtain the potential of the sink-flow, from which the desired radial velocity components are to be defined. With κ as the strength of the sink per unit of surface, and R as the distance of an elementary sink dF from starting point P with the coordinates z and r, the potential $d\Phi$ for this point is

$$d\Phi = \frac{\kappa \, dF}{R} \tag{3}$$

With the introduction of the angle θ between the radii

*There are no fundamental difficulties preventing this method of representation from being extended to include the case of nonuniform thrust grading over the propeller disk area.

r and r_o as a further variable, the area of the elementary sink at distance r_o from $dF = r_o\, dr_o\, d\theta$. Besides, as $R = \sqrt{r_o^2 + r^2 - 2r\, r_o \cos\theta + z^2}$, the potential Φ of the evenly covered propeller disk area is:

$$\Phi = \kappa \int_{\theta=0}^{2\pi} \int_{r_o=0}^{a} \frac{r_o\, dr_o\, d\theta}{\sqrt{r_o^2 + r^2 - 2r\, r_o \cos\theta + z^2}} \qquad (4)$$

The strength of the sink being already defined by v_a, the quantity κ will have to be expressed through v_a.

For further analysis of Φ, we simply integrate over the radius of the propeller disk from $r_o = 0$ to $r_o = a$ and differentiate the thus obtained term, according to r. (See Appendix.) The result is the radial velocity component w_r at point P in form of

$$w_r = \frac{\partial\theta}{\partial r} = -2\,\kappa\, a \int_0^{\pi} \frac{\cos\theta\, d\theta}{\sqrt{a^2 + r^2 - 2\,a\,r\cos\theta + z^2}} \qquad (5)$$

This integral is reducible to two normal perfectly elliptical integrals, which finally give (see Appendix):

$$w_r = \frac{2\kappa}{r}\sqrt{(a+r)^2 + z^2}\left[E\left(k, \frac{\pi}{2}\right) - \frac{a^2 + r^2 + z^2}{(a+r)^2 + z^2} K\left(k, \frac{\pi}{2}\right)\right] \qquad (6)$$

K and E are the complete elliptical integrals of first and second order. The value of modulus k is

$$k = \frac{2\sqrt{a\,r}}{\sqrt{(a+r)^2 + z^2}} \qquad (7)$$

Since the fluid is to enter normal to the propeller disk area at speed $v_a/2$, quantity κ, which defines the strength of the sink-flow, must be expressed by v_a. To this end we visualize a sphere of radius R placed around an elementary sink having a surface expansion of $dF = r_o\, dr_o\, d\theta$. The extent of the sink is infinitely small, hence its effect on points at finite distance is the same as a point sink. On the basis of the continuity, this means that the fluid quantity which disappears in the sink must enter through the sphere of radius R at identical speed dw_R normal to the surface of the sphere. This speed is according to equation (3):

$$dw_R = - \frac{\kappa\, dF}{R^2} \tag{8}$$

The condition that the volume of fluid sucked in per second by the elementary sink dF must equal the quantity passing through the surface of the sphere, affords

$$4\, R^2\, \pi\, dw_R = 2\, dF\, \frac{v_a}{2} \tag{9}$$

The right-hand side of (9) gives the quantity per second disappearing in the sink. The area is $2\, dF$ because the fluid enters on either side at speed $v_a/2$. Writing dw_R in (9) conformable to (8) gives κ as

$$\kappa = - \frac{v_a}{4\pi} \tag{10}$$

This defines the radial component of the velocity w_r at any point of the velocity field. The vertical component w, necessary for computing the change of induced drag, is then readily obtainable. With the notations of figure 4, $FD = h$, $DB = x$ and $\sphericalangle\, DBF = \epsilon$, it is

$$w = w_r \sin \epsilon = w_r \frac{h}{\sqrt{h^2 + x^2}} \tag{11}$$

and the vertical velocity w along the wing span amounts to

$$w = \frac{v_a}{2} \frac{\sin \epsilon}{r\, \pi} \sqrt{(a+r)^2 + z^2}\, \left[E\left(k, \frac{\pi}{2}\right) - \frac{a^2 + r^2 + z^2}{(a+r)^2 + z^2}\, K\left(k, \frac{\pi}{2}\right)\right] \tag{12}$$

Figure 5 shows this speed w for a specific case.

For the computation of the induced drag change we abbreviate this formula to read:

$$w = G\, v_a \tag{13}$$

With $\frac{b}{2}$ = semispan of wing, the change of induced drag ΔW_i is expressed by the well-known integral:

$$\Delta W_i = \int_{-\frac{b}{2}}^{+\frac{b}{2}} \frac{\Delta W}{V} \, d\, A \tag{14}$$

or

$$\Delta W_i = \int_{-\frac{b}{2}}^{+\frac{b}{2}} \frac{G \; v_a \; dA}{V} \tag{15}$$

Since v_a is constant and expressible by the load factor and the speed V conformably to the slipstream theory with

$$v_a = V \left(\sqrt{1 + c_s} - 1\right) \tag{16}$$

it is

$$\Delta W_i = \left(\sqrt{1 + c_s} - 1\right) \int_{-\frac{b}{2}}^{+\frac{b}{2}} G \; d\; A \tag{17}$$

The lift distribution across the span is assumed elliptical and the lift dA of an element is expressed as lift coefficient c_a. A brief calculation yields

$$dA = \frac{4}{\pi} c_a \; q \; t \sqrt{1 - \left(\frac{x}{b/2}\right)^2} \; dx \tag{18}$$

wherein q = dynamic pressure and t = wing chord. With this value, we have:

$$\Delta W_i = \frac{4}{\pi} c_a \; q \; t \; \left(\sqrt{1 + c_s} - 1\right) \int_{-\frac{b}{2}}^{+\frac{b}{2}} G \sqrt{1 - \left(\frac{x}{b/2}\right)^2} \; dx \tag{19}$$

and the change in drag coefficient

$$c_{w_i} = \frac{4 c_a}{\pi b} \left(\sqrt{1 + c_s} - 1\right) \int_{-\frac{b}{2}}^{+\frac{b}{2}} G \sqrt{1 - \left(\frac{x}{b/2}\right)^2} \; dx \tag{20}$$

This integral must be evaluated graphically or mathematically.

IV

In order to compare the theoretical with the experimental results, we resorted to the previously mentioned experiments (reference 1) and specifically to two cases where the wing was outside of the propeller slipstream, so as to retain the validity of our results. In one case the propeller was below, in the other, above the wing. The experimental results and further details on the mutual location are tabulated in tables Nos. 123 and 127 of the Göttingen test report.

The calculated change of induced drag Δc_{w_i} is in close accord with the test data. Figure 6 shows the polar of the uninfluenced wing as solid curve, while the two dashed polar curves were computed from it. The polar a is valid for the case of propeller above, and polar b for propeller below the wing. The test points indicated as small circles manifest a quite close agreement, particularly for case a.

As concerns the absolute magnitude of Δc_{w_i}, it is seen to be about 10 percent of the wing drag. This figure should be even lower for level flight in most practical cases, since the propeller thrust in this particular case is comparatively high compared to the wing drag. Obviously, Δc_{w_i} becomes smaller as the thrust decreases.

APPENDIX

Analysis of Velocity Component w_r from the Potential of the Propeller Disk Area Superposed with Evenly Distributed Vortex Sinks

The potential Φ/κ of the propeller disk area* evenly covered with sinks is, according to equation (4):

$$\frac{\Phi}{\kappa} = \int_{\theta=0}^{2\pi}\int_{r_0=0}^{a} \frac{r_0 dr_0 d\theta}{R} = \int_{\theta=0}^{2\pi}\int_{r_0=0}^{a} \frac{r_0 dr_0 d\theta}{\sqrt{r_0^2+r^2-2r\ r_0 \cos\theta+z^2}}$$

The term below the root may equally be written as

$$r_0^2+r^2-2r\ r_0 \cos\theta+z^2=(r_0-r\cos\theta)^2+r^2\sin^2\theta+z^2=R^2$$

The addition of $r\cos\theta\ dr_0\ d\theta$ to the numerator of the fraction followed by subtraction gives the partial integrals:

$$\frac{\Phi}{\kappa} = \int_0^{2\pi} d\theta \int_0^a \frac{(r_0-r\cos\theta)\ dr_0}{R} + r\int_0^{2\pi}\cos\theta\ d\theta\int_0^a \frac{dr_0}{R}$$

Integration over the radius of the propeller disk in conjunction with the insertion of limits and the abbreviation $a^2 + r^2 - 2a\ r\cos\theta + z^2 = R_a^2$ yields:

$$\frac{\Phi}{\kappa} = \int_0^{2\pi} R_a\ d\theta - 2\pi\sqrt{r^2+z^2} + r\int_0^{2\pi}\cos\theta\ \ln(R_a+a-r\cos\theta)d\theta$$

$$-\int_0^{2\pi}\cos\theta\ \ln(\sqrt{r^2+z^2}-r\cos\theta)d\theta.$$

The component w_r of the velocity along r is obtained by differentiation of the potential according to r, re-

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*In a previous report (Wieselsberger: The Influence of a Propeller on a Wall, Aachen Reports Nos. 10), devoted to the velocity field surrounding a propeller, the potential was expressed by Bessel's functions. The much more simple methods given here, were indicated by O. Blumenthal.

sulting in:

$$\frac{w_r}{\kappa} = \frac{1}{\kappa}\frac{\partial\Phi}{\partial r} = \int_0^{2\pi} \frac{(r-a\cos\theta)d\theta}{R_a} - \frac{2\pi r}{\sqrt{r^2+z^2}}$$

$$+ \int_0^{2\pi} \cos\theta \ln(R_a+a-r\cos\theta)d\theta$$

$$+ r\int_0^{2\pi} \frac{\cos\theta}{R_a+a-r\cos\theta}\left(\frac{r-a\cos\theta}{R_a}\right)d\theta$$

$$- \int_0^{2\pi} \cos\theta \ln\left(\sqrt{r^2+z^2}-r\cos\theta\right)d\theta$$

$$- r\int_0^{2\pi} \frac{\cos\theta}{\sqrt{r^2+z^2}-r\cos\theta}\left(\frac{r}{\sqrt{r^2+z^2}} - \cos\theta\right)d\theta,$$

which reduces to

$$w_r = -2\kappa a\int_0^{\pi} \frac{\cos\theta\, d\theta}{R_a} = -2\kappa a\int_0^{\pi} \frac{\cos\theta\, d\theta}{\sqrt{a^2+r^2-2a r\cos\theta+z^2}}$$

Then we abbreviate:

$$a^2 + r^2 + z^2 = m^2,$$

divide the integral by adding $m^2\, d\theta$ to the numerator and then subtract, which gives:

$$w_r = \frac{\kappa}{r}\int_0^{\pi} \frac{-2a r\cos\theta\, d\theta}{\sqrt{m^2-2a r\cos\theta}} = \frac{\kappa}{r}\int_0^{\pi} \frac{m^2-2a r\cos\theta - m^2}{\sqrt{m^2-2a r\cos\theta}}d\theta$$

$$= \frac{\kappa}{r}\int_0^{\pi} \sqrt{m^2-2a r\cos\theta}\, d\theta - \frac{\kappa m^2}{r}\int_0^{\pi} \frac{d\theta}{\sqrt{m^2-2a r\cos\theta}}$$

Then we put $\theta = \pi - \varphi$ and reverse the limits because $\varphi = \pi$ for $\theta = 0$, and $\varphi = 0$ for $\theta = \pi$. Besides, $d\theta = - d\varphi$. The result is:

$$w_r = -\frac{\kappa}{\pi}\int_{\pi}^{0}\sqrt{m^2+2a\ r\ \cos\varphi}\ d\varphi + \frac{\pi m^2}{r}\int_{\pi}^{0}\frac{d\varphi}{\sqrt{m^2+2a\ r\ \cos\varphi}}$$

By reversing the prefixes of the two summands, the original limits may then be used again. Concurrently, we introduce the half angle by utilizing the elementary trigonometrical relation,

$$\cos\varphi = 1 - 2\sin^2\frac{\varphi}{2}.$$

We obtain:

$$w_r = \frac{\kappa}{r}\int_0^{\pi}\sqrt{m^2 + 2a\ r\left(1 - 2\sin^2\frac{\varphi}{2}\right)}\ d\varphi$$

$$- \frac{\kappa m^2}{r}\int_0^{\pi}\frac{d\varphi}{\sqrt{m^2 + 2a\ r\left(1 - 2\sin^2\frac{\varphi}{2}\right)}}$$

and, when writing the value for m^2:

$$w_r = \frac{\kappa}{r}\sqrt{(a+r)^2 + z^2}\left[\int_0^{\pi}\sqrt{1 - k^2\sin^2\frac{\varphi}{2}}\ d\varphi\right.$$

$$\left. - \frac{a^2 + r^2 + z^2}{(a + r^2) + z^2}\int_0^{\pi}\frac{d\varphi}{\sqrt{1 - k^2\sin^2\frac{\varphi}{2}}}\right]$$

where the modulus k of both elliptical integrals has the value,

$$k = \frac{2\sqrt{a\ r}}{\sqrt{(a+r)^2 + z^2}}$$

Writing, finally, $\frac{\varphi}{2} = \theta$, the new limits have the values $\theta = 0$ and $\theta = \frac{\pi}{2}$; and w_r becomes the ultimate value given in equation (6):

$$w_r = \frac{2\kappa}{r}\sqrt{(a+r)^2+z^2}\left[E\left(k, \frac{\pi}{2}\right) - \frac{a^2+r^2+z^2}{(a+r)^2+z^2}K\left(k, \frac{\pi}{2}\right)\right]$$

REFERENCES

1. Prandtl, L.: Gegenseitige Beeinflussung von Tragflügel und Schraube. Ergebn. der Aerodyn. Versuchsanstalt zu Göttingen. Report I, 1921.

2. Betz, A.: Considerations on Propeller Efficiency. T.M. No. 481, N.A.C.A., 1928.

3. Helmbold, H. B.: Über die Wechselwirkung zwischen Luftschraube und Flugzeug. Verhandl. des 5, Internat. Luftfahrtkongresses im Haag, 1930, p. 575.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

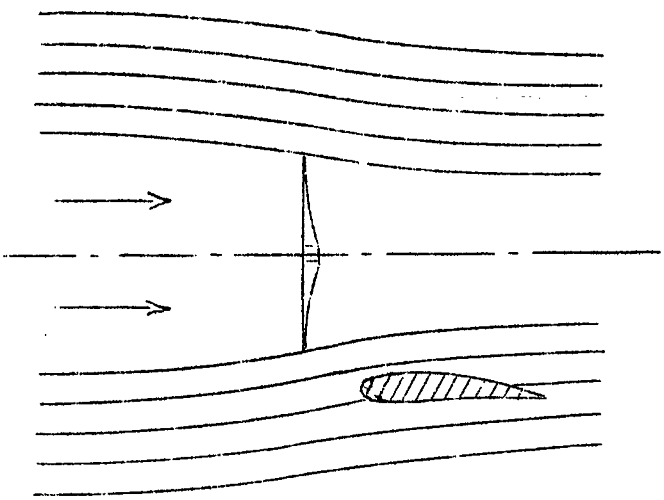

Figure 1.-Mutual interference of wing and propeller.

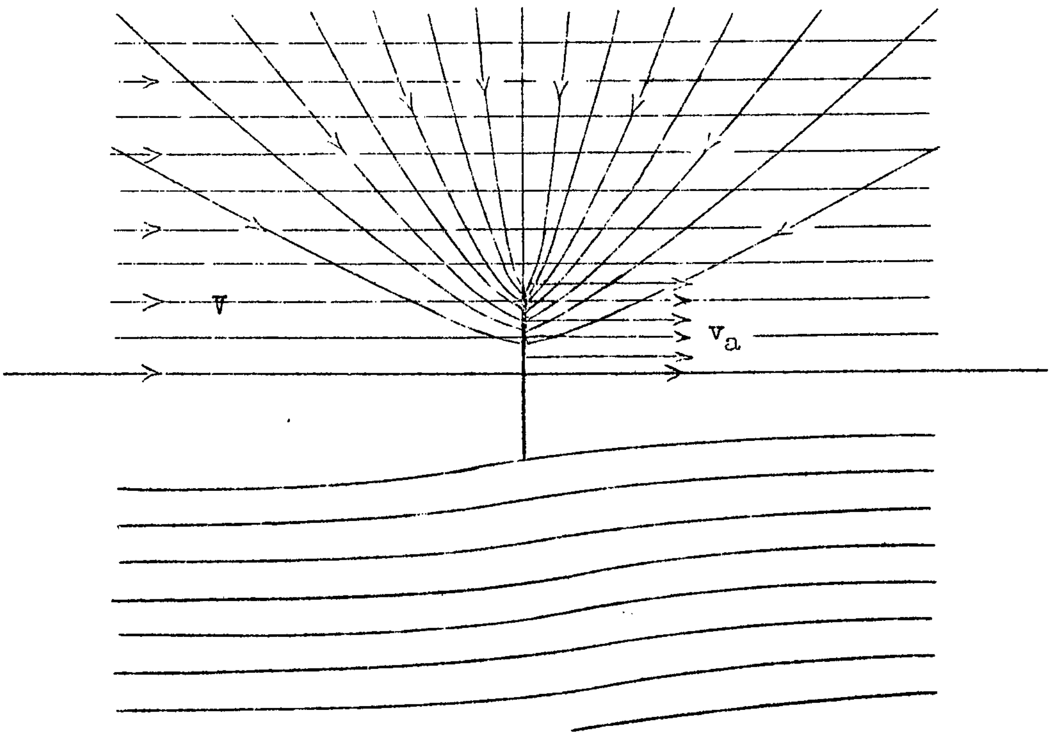

Figure 2.-Propeller flow visualized as superposed partial flows.

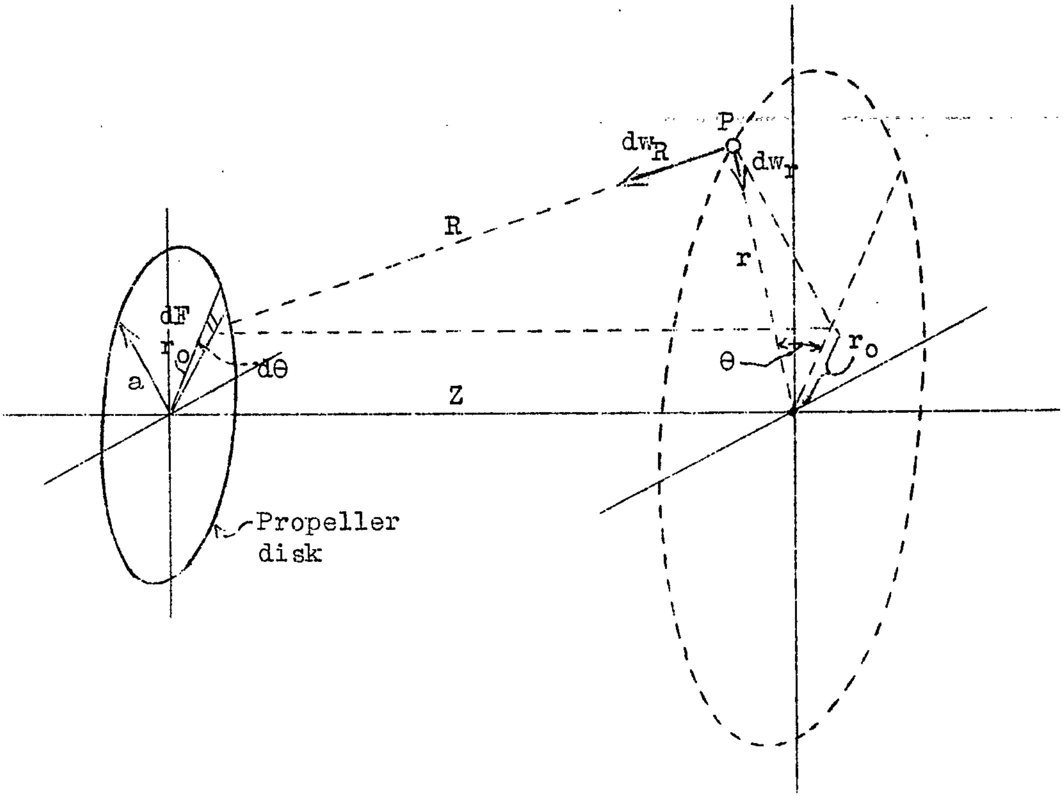

Figure 3.-Analysis of potential of circular surface covered evenly with vortex sinks.

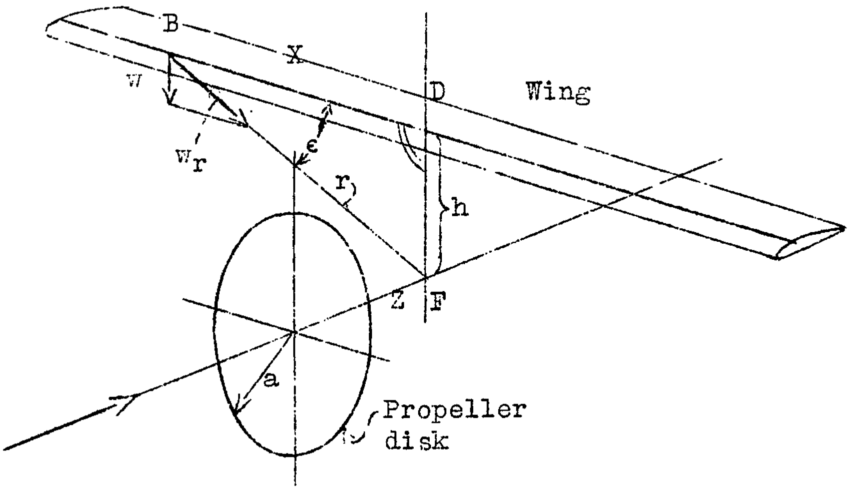

Figure 4.-Induced drag due to propeller.

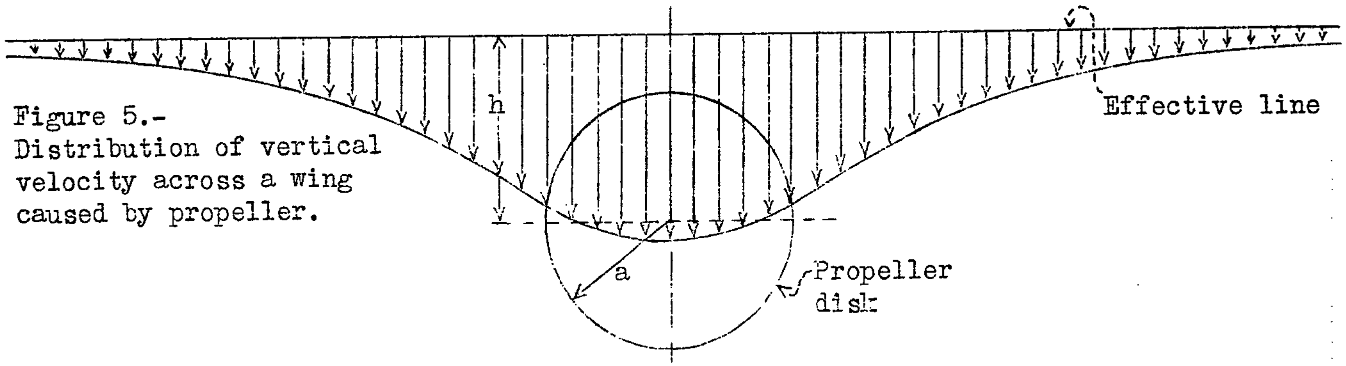

Figure 5.- Distribution of vertical velocity across a wing caused by propeller.

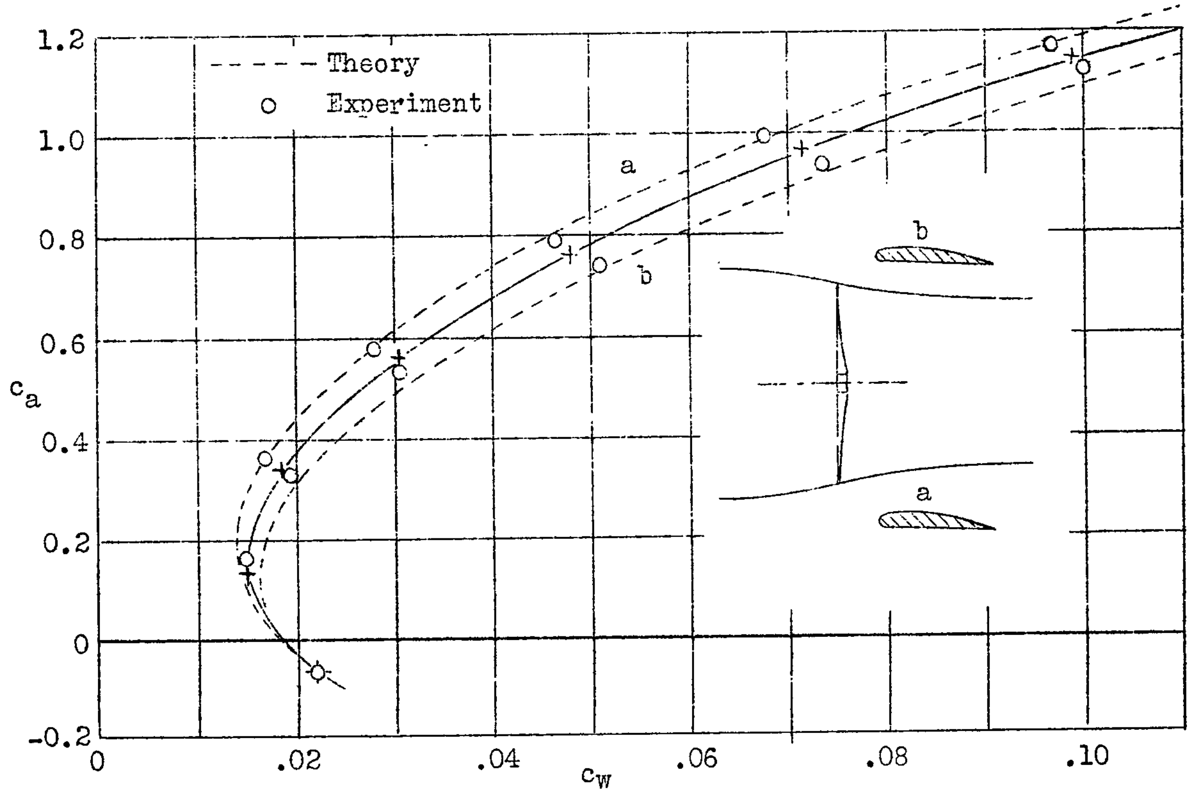

Figure 6.- Comparison of experimental and extrapolated polar curve.